Real World Data

I. UNIT OVERVIEW & PURPOSE:

This unit will be about Real World Data and the graphs that can be created from them. The students will either use given data, gather their own data via surveys, etc., or use internet sources to collect data. In the five connected lessons in this unit, the students will then create various graphs using Excel, including linear, exponential, logarithmic, and quadratic. Students will use Excel to find lines and curves of best fit for the data as well as being able to answer questions about their graphs and to predict future outcomes given their lines and curves of best fit.

II. UNIT AUTHOR:

Melanie Haupt, Auburn High School, Riner VA
Robin Malone, Patrick Henry High School, Ashland VA

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Data Analysis

V. OBJECTIVES:

The students will use various collection techniques to collect data for a real-world problem which will be presented to them. The students will be able to enter and manipulate the data in a spreadsheet (Excel). The students will use their data to create scatter plots with linear, quadratic, exponential, trigonometric and logarithmic tendencies.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.2 Students will collect and analyze data, determine the equation of the curve of best fit, make predictions and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.9.The student will design and conduct an experiment/survey. Key concepts include

a) sample size;
b) sampling technique;
c) controlling sources of bias and experimental error;
d) data collection; and
e) data analysis and reporting.

MPE.12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14.The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

MPE.22The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

VII. CONTENT:

This unit addresses real world examples of the use of spreadsheets, finding equations of curves of best fit.

VIII. REFERENCE/RESOURCE MATERIALS:

The students will use computer spreadsheet software, like Microsoft Excel to organize the data and to create their various graphs. The students will also research websites on the internet for data for at least one lesson.

IX. PRIMARY ASSESSMENT STRATEGIES:

Classroom activities, journal entries, finished spreadsheets with graphs, answers to questions about their data collection and graphs.

X. EVALUATION CRITERIA:

The classroom activities will have worksheets to facilitate the research portions of the units. These will be graded for completeness and accuracy. The rubrics for the journal entries and spreadsheets will be attached to each relevant lesson.

XI. INSTRUCTIONAL TIME:

6-90 minute blocks.

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Lesson 1 My Favorite Meal

Strand
Data Analysis

Mathematical Objective(s)
Students will use specific websites to collect nutrition data from various restaurants. Students will then enter data into a spreadsheet, as well as create and use formulas in the spreadsheet. Students will create 3 charts for this data in the spreadsheet. Students will then use this information to answer questions about the data and their charts.

Mathematics Performance Expectation(s)
MPE.2 Students will collect and analyze data, determine the equation of the curve of best fit, make predictions and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.22 The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

Related SOL

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

PH.2 The student will investigate and understand how to analyze and interpret data. Key concepts include

a) a description of a physical problem is translated into a mathematical statement in order to find a solution;
b) relationships between physical quantities are determined using the shape of a curve passing through experimentally obtained data;
c) the slope of a linear relationship is calculated and includes appropriate units;
d) interpolated, extrapolated, and analyzed trends are used to make predictions;
e) situations with vector quantities are analyzed utilizing trigonometric or graphical methods

NCTM Standards

- Understand histograms, parallel box plots, and scatterplots and use them to display data
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
- Understand relations and functions and select, convert flexibly among, and use various representations for them

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- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Lesson requires computers connected to the internet and loaded with a spreadsheet program such as Excel.
- "Spreadsheet and Charts" Handout.
- "My Favorite Meals" Handout.
- "Food Pyramid" Handout.

Assumption of Prior Knowledge

- Students should already be able to figure out basic formulas such as SUM and AVERAGE.
- Students should understand percentages.
- Students should have a basic understanding of how to enter data into a spreadsheet.
- Students should be operating on Deduction level on Van Hiele scale with respect to data analysis.
- This lesson draws on the important social issue of nutrition, exploring foods students may already eat, as well as looking at what types of foods are recommended by the USDA.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will explore the nutritional content of various restaurants, and then enter and manipulate the data in a spreadsheet, and create charts from the data.
- The lesson should take one 90-minute block.
- To introduce the lesson, ask your students to write down their favorite restaurant, and their favorite meal from there.
- The teacher should decide if the students will be working individually or in pairs, and break into groups accordingly.
- At the end of the activity, the students should write their meal and the calorie count and fat percentage on the "Favorite Meal Poster" so the results can be shared with the class.

Student Exploration:

- **Individual Work or Small Group Work:** The teacher should decide if the students will be working individually or in pairs.
- **Whole Class Sharing:** At the beginning of the activity, the class should briefly discuss what options for restaurants they will have for the lesson.
- **Whole Class Sharing:** At the end of the activity, the students should write their meal and the calorie count and fat percentage on the "Favorite Meal Poster" so the results can be shared with the class.

Student/Teacher Actions:

- Each student or group of students should have a computer and the Handouts.
- Students should begin by choosing their restaurant and going to their website. Then they should follow the directions on the Handouts, "Spreadsheets and Charts" and "My Favorite Meal".

- The teacher should walk around the room, monitoring student progress, and helping when needed. Depending on the level of knowledge of spreadsheets, some students may initially require more guidance than others.

Monitoring Student Responses

- Students should be discussing what items they are choosing for their meal. They should be assisting each other as they start to organize and set up their spreadsheets. Students are required to come up with several formulas on their own, or to be able to use the formulas already in the software. The teacher may need to help students who are struggling with this, but this will enable the students to complete future lessons with less guidance.

Assessment

- Should the teacher choose, they could assign a journal entry asking the students to write about two to three things they learned during the lesson.
- The spreadsheet itself, along with the charts, should be printed out to be graded by the teacher. There is a rubric attached for grading the spreadsheets.
- The students will also put their meal information on the class poster, so that ALL the data can be looked at for patterns.

Extensions and Connections (for all students)

- Is there a relationship between nutrition and health? Can this help to encourage a healthier life style?
- This lesson is one of a Unit of five lessons dealing with Real-World data, its analysis and graphical representations.

Strategies for Differentiation

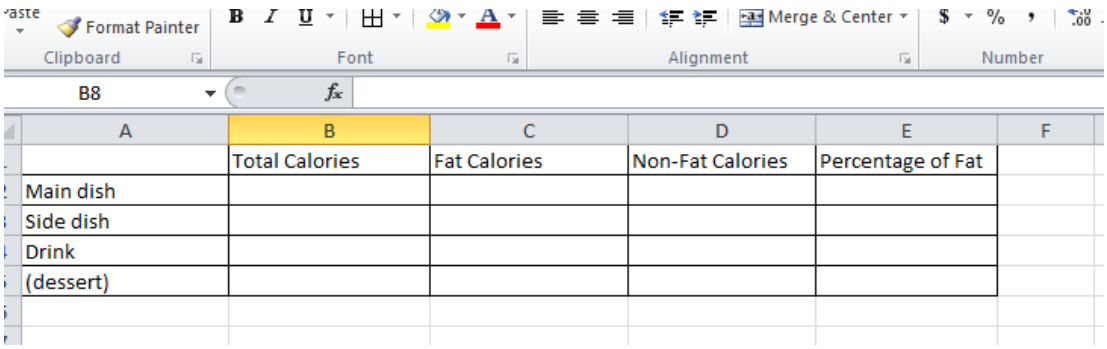
- Students are using various skills in this lesson, including visual and kinesthetic, and using computers. Students with reading disorders could have another student (their partner) read them the instructions. They could also share the responsibility for researching on the internet. For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student exploration questions.
- Visual learners will benefit from the graphical representations within all the lessons.
- Auditory learners with benefit from the classroom and group discussions.
- High ability students may start to pick up on key features when creating graphs and to decide on what an appropriate model would be.

Spreadsheets & Charts Name_____Date_____

1. Pick your favorite restaurant and go to their website – they must have nutritional data available.

2. Once you have chosen your restaurant, you will choose a menu for the whole day – Breakfast, Lunch and Dinner. Feel free to mix restaurants. Each meal should consist of a main dish, a side dish and a drink. Dinner should include dessert.

3. For each item, record, the TOTAL CALORIES and the CALORIES FROM FAT in your spreadsheet. Below is a SAMPLE of what your spreadsheet might look like – customize it for yourself!



	A	B	C	D	E	F
		Total Calories	Fat Calories	Non-Fat Calories	Percentage of Fat	
	Main dish					
	Side dish					
	Drink					
	(dessert)					

4. You will need to figure out a FORMULA to find the Non-Fat Calories and you will need a FORMULA to figure out the Percentage of Fat for each item.

5. Sum each COLUMN using a formula.

6. Make sure you have formatted your cells to "wrap text" and to keep the numbers whole – no decimal places.

7. You will need to create THREE different types of charts from your data.

   - Make a PIE CHART for your BREAKFAST meal of the types of food and their corresponding Percentage of Fat. Title this graph "Percentage of Fat per Breakfast Food Item"
   - Make a COLUMN GRAPH for your LUNCH meal of the types of food, and both the Fat Calories and the Non-Fat Calories. Title this graph "Fat vs. Non-Fat Calories in Lunch.
   - Make a SCATTER PLOT for your DINNER of Total Calories vs. Percentage of Fat. Title this graph "Total Calories vs. Percentage of Fat for Dinner".

8. Once you're done with these directions, PRINT your page and put your Calorie Info on the Poster.

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PICK ONE OF YOUR MEALS TO ANSWER THESE QUESTIONS.

Answer the following questions using your spreadsheet and charts. Do not fill in the shaded boxes.

Total Calories for the Meal			
Total Fat Calories for the Meal			
Total Non-Fat Calories for the Meal			
Percentage of Fat for Meal			
Item with MOST Calories		Number of Calories for the item	
Item with LEAST Calories		Number of Calories for the item	
Item with HIGHEST Percentage of Fat		Calories for this item	
Item with LOWEST Percentage of Fat		Calories for this item	

OKAY – so what does all this mean?

1. Were you at all surprised by the number of Calories in your favorite meal? Explain.

2. What relevance, if any, is there between the Total Calories of an item and its Percentage of Fat? Back up your answer with facts from your spreadsheet.

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Look at the Handout "The Food Pyramid"

3. How many Calories does ONE gram of carbohydrates have?

4. How many Calories does ONE gram of fat have?

5. What percentage of Fat should there be in your diet?

In a spreadsheet, create a worksheet where you can total each of the following categories:

Fat, Saturated Fats, Cholesterol, Sodium, Potassium, Carbohydrates, Dietary Fiber, Protein

Use a spreadsheet formula to calculate the total amounts for each of these categories.

Make sure you print out the spreadsheet to turn in.

Category	Recommended amount	Your amount
Fat		
Saturated Fat		
Cholesterol		
Sodium		
Potassium		
Carbohydrates		
Dietary Fiber		
Protein		

6. How did your meal compare to the recommended daily allowances?

7. What impact, if any, did this lesson have on your thoughts about your favorite food? Explain.

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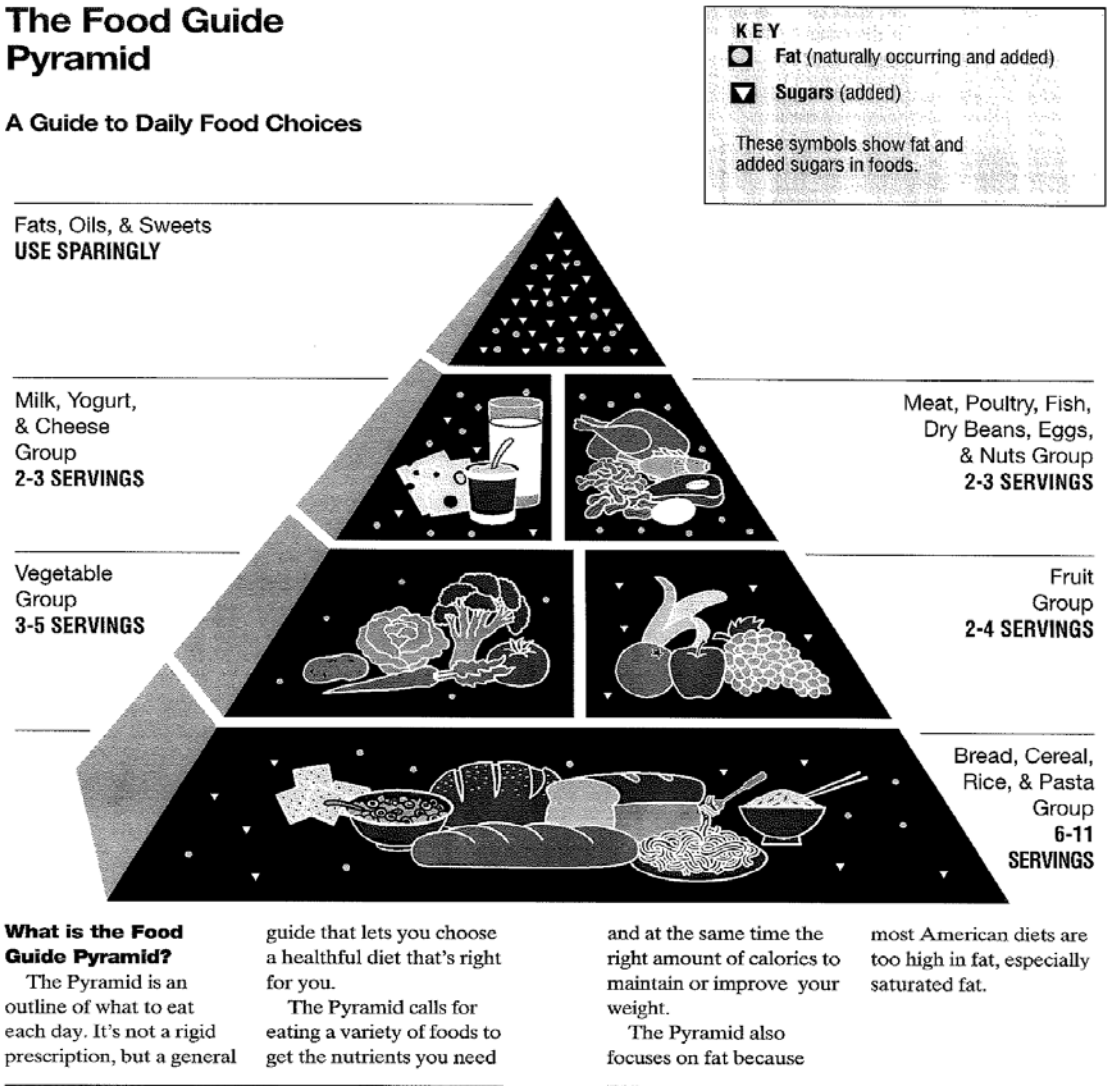

The following table lists the DVs based on a calorific intake of 2,000 calories, for adults and children four or more years of age.

Total Fat	65 g
Saturated Fatty Acids	20 g
Cholesterol	300 mg
Sodium	2400 mg
Potassium	4700 mg
Total Carbohydrate	300 g
Dietary Fiber	25 g
Protein	50 g

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Name_____ Date____

Grading Rubric for Spreadsheets & Charts

	C	B	A
Enter and format data; create and use formulas.	Entered data correctly, formatting mostly correct, attempted to use formulas.	Entered data correctly, formatting correct, used formulas correctly.	Entered data correctly, used formulas correctly. Made the spreadsheet look AWESOME!! (advanced formatting, etc.)
Create a Pie Chart	Minimal chart, correct data displayed.	Correct data displayed, included title, legend and data labels.	Correct data displayed, included title, legend and data labels. Made the chart look AWESOME!! (advanced formatting, etc.)
Create a Column Graph	Minimal chart, correct data displayed.	Correct data displayed, included title, legend and data labels.	Correct data displayed, included title, legend and data labels. Made the chart look AWESOME!! (advanced formatting, etc.)
Create a scatter plot; add trendline to find the equation of the curve of best fit; find the r-squared value.	Minimal chart, correct data displayed. Trendline incorrect.	Chart correct, correct data displayed, Trendline correct, but r-squared value not shown.	Chart correct, correct data displayed, Trendline correct and r-squared value shown.

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Lesson 2 Motion in Free Fall

Strand
Data Analysis

Mathematical Objective(s)
Students will use motion detectors to assist them in collecting data. Students will then enter data into a spreadsheet and create an equation to represent the data. Students will then answer questions about the activity.

Mathematics Performance Expectation(s)
MPE.2 Students will collect and analyze data, determine the equation of the curve of best fit, make predictions and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

PH.4 The student will investigate and understand how applications of physics affect the world. Key concepts include
a) examples from the real world; and
b) exploration of the roles and contributions of science and technology.

PH.2 The student will investigate and understand how to analyze and interpret data. Key concepts include

a) a description of a physical problem is translated into a mathematical statement in order to find a solution;
b) relationships between physical quantities are determined using the shape of a curve passing through experimentally obtained data;
c) the slope of a linear relationship is calculated and includes appropriate units;
d) interpolated, extrapolated, and analyzed trends are used to make predictions;
e) situations with vector quantities are analyzed utilizing trigonometric or graphical methods

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NCTM Standards

- Understand histograms, parallel box plots, and scatterplots and use them to display data
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

Describe the materials and resources (including instructional technology) you plan to use in each lesson.

- Computers with Excel
- Motion Detectors with CBL's
- Handout "Free Fall and Parabolas."

Assumption of Prior Knowledge

- Students should already be able to use a motion detector and CBL.
- Students should have a basic understanding of how to enter data into a spreadsheet.
- The students should be very familiar with parabolas, including the different equation forms.
- Students should be operating on Deduction level on Van Hiele scale with respect to data analysis.

Introduction: Setting Up the Mathematical Task

- The lesson should take one 90-minute block.
- In this lesson, students will investigate free fall and the fact that motion in free fall is parabolic. Students will collect data by using motion detectors to record their height vs. time as they jump up into the air and come back down.
- To introduce the lesson, show the video clip from October Sky. Explain a bit of the background, saying that the movie is about a boy who was experimenting with model rockets and their paths of motion.
- Here is a link to the video clip.
  October Sky video clip
- Now explain that you can't really use rockets like they did in the movie, but you are going to modify the experiment by using the motion of a standing jump.
- The teacher should break the students up into groups of three. Hand out the equipment, making sure each group of students understands how the equipment works.
- At the end of the activity, the students will answer the questions on the Handout "Free Fall and Parabolas".

Student Exploration:

Small Group Work: In groups of three, students should log into their computer, and make sure their motion detector and CBL (or direct link to the computer) is set up and working. Students will take turns performing a standing jump in front of the motion detector. Record the data.

Student/Teacher Actions:

- Each student or group of students should have a computer, a motion detector, a CBL if needed and the Handouts "Free Fall and Parabolas".
- Students should begin by logging into their computer and connecting their motion detectors and CBL's.
- Students should open Excel and set up their spreadsheets.
- The teacher should walk around the room, monitoring student progress, and helping when needed. Depending on the level of knowledge of motion detectors and CBL's, some students may initially require more guidance than others.

Monitoring Student Responses

- Students should be discussing any problems they are having with their equipment. They should be assisting each other as they start to organize and set up their spreadsheets. Students may need to download the data from the motion detector if a CBL is used. The teacher may need to help students who are struggling with this, but this will enable the students to complete future lessons with less guidance.

Assessment

- Should the teacher choose, they could assign a journal entry asking the students to write about two to three things they learned during the lesson.
- The spreadsheet itself, along with the charts, should be printed out to be graded by the teacher. There is a rubric attached for grading the spreadsheets.
- The students will also answer the questions on the Handout "Free Fall and Parabolas".

Extensions and Connections (for all students)

- This lesson is directly related to physics and the study of kinematics and the motion of freely falling objects.
- Parabolic motion is seen every day – in nearly every game that is played with a ball, for example.
- This lesson is one of a Unit of five lessons dealing with Real-World data, its analysis and graphical representations.

Strategies for Differentiation

- Students are using various skills in this lesson, including visual and kinesthetic, and using computers. Students with reading disorders could have another student (their partner) read them the instructions. They could also share the responsibility for researching on the internet. For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.

- Learning disabled students may benefit if the teacher provides multiple choice answers to the student exploration questions.
- Visual learners will benefit from the graphical representations within all the lessons.
- Auditory learners with benefit from the classroom and group discussions.
- High ability students may start to pick up on key features when creating graphs and to decide on what an appropriate model would be.

Free Fall and Parabolas Name_____Date__

First of all – you will need to answer the question WHAT IS FREE FALL?? Discuss with your classmates (some of them may have taken physics), look it up on the internet or ask your teacher. Then write the answer here!

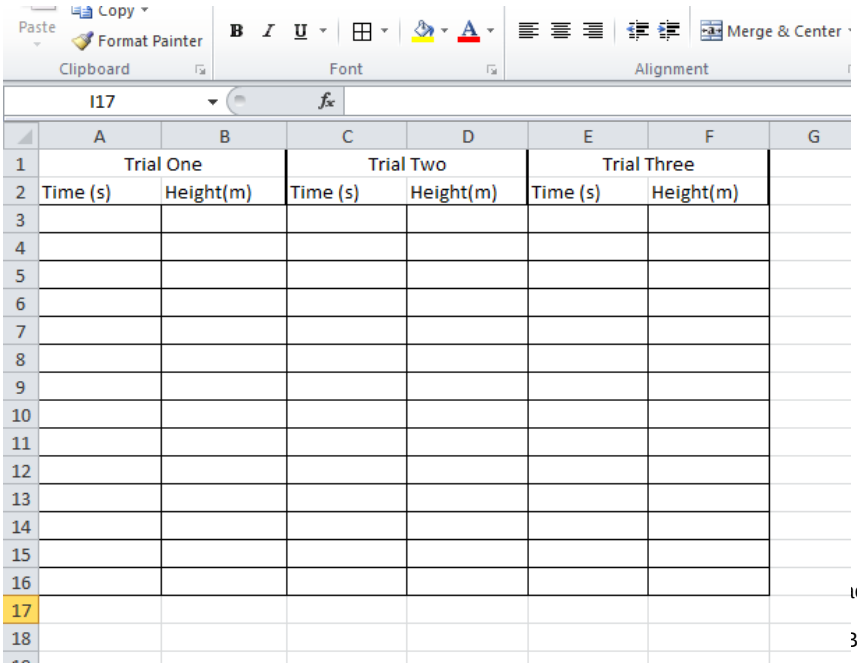
You will need a group of at least three people.

Materials Needed

- Motion detector and CBL
- Class set of balls – bouncy, tennis
- Meter stick
- Yourself

Part 1 Data Collection

1. Connect the CBL and motion detector.
2. You will take turns jumping straight up into the air in front of the motion detector and record your data.
3. Do ONE trial for EACH person and record your data in an excel spreadsheet. (Sample below)
   (If your trial has some problems, try again and "write over" the original data.)



	A	B	C	D	E	F	G
1	Trial One		Trial Two		Trial Three		
2	Time (s)	Height(m)	Time (s)	Height(m)	Time (s)	Height(m)	
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							

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4. For each trial, select TIME to be the independent variable and HEIGHT to be the dependent variable.

5. Make a scatter plot for each trial – make sure to label the trials, one, two and three (or your names)

6. Now add a TRENDLINE. What type of curve do you think will be a best fit for the data?

7. Make sure to click the boxes to "display equation on chart" and "display r-squared value on chart".

Part 2 Answer the following questions using ONE OF YOUR GRAPHS and your data:

1. What type of graph was a best fit match for your data? _____

2. What was the equation of the curve of best fit? _____

3. Do you feel confident that this line is the best match for your data. _____

   Why or why not_____

4. What are the coordinates of the maximum data point on the graph? _____

5. What are the coordinates of the minimum data point on the graph? _____

6. Based on your TYPE of graph, is there another TERM you could use to describe the maximum data point? _____

7. What are the key features of the graph? _____

   _____

8. Write the equation of the curve of best fit in a different FORM. _____

9. Verify that the equation from #2 and the equation from #7 are equal.

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Part 3 More data collection

1. The motion detector and CBL should still be set up from part 1.
2. Using the meter stick and a piece of tape, mark a spot of a wall 2m up.
3. Drop the ball in front of the motion detector from the height of the tape mark.
4. Do three trials for dropping the the ball.
5. For each trial, select TIME to be the independent variable and HEIGHT to be the dependent variable.
6. Make a scatter plot for each trial – make sure to label the trials, one, two and three.
8. Now add a TRENDLINE. What type of curve do you think will be a best fit for the data?
9. Make sure to click the boxes to "display equation on chart" and "display r-squared value on chart".

Part 4 Answer the following questions based on the bouncing ball.

1. Compare the graphical results of the bouncing ball to the jumping from part 1. Describe any similarities and/or differences.

2. Why do you think the graphs are parabolic in shape?

Now let's try a FUN example of the application of parabolas and free fall!!

Go to http://www.teachmathematics.net/activities/angry-birds.htm

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Lesson 3 Cooling Temperatures

Strand
Data Analysis

Mathematical Objective(s)
The students will collect, graph, and analyze data. They will choose an appropriate mathematical model for their data. They will also make predictions based on their model.

Mathematics Performance Expectation(s)
MPE.2. Students will collect and analyze data, determine the equation of the curve of best fit, make predictions and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing with be employed.

Related SOL

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

PH.4 The student will investigate and understand how applications of physics affect the world. Key concepts include
a) examples from the real world; and
b) exploration of the roles and contributions of science and technology.

PH.2 The student will investigate and understand how to analyze and interpret data. Key concepts include

a) a description of a physical problem is translated into a mathematical statement in order to find a solution;
b) relationships between physical quantities are determined using the shape of a curve passing through experimentally obtained data;
c) the slope of a linear relationship is calculated and includes appropriate units;
d) interpolated, extrapolated, and analyzed trends are used to make predictions;
e) situations with vector quantities are analyzed utilizing trigonometric or graphical methods

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NCTM Standards

- Understand histograms, parallel box plots, and scatterplots and use them to display data
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Lesson requires computers connected to the Internet and loaded with a spreadsheet program such as Excel.
- Cups of varying types with very hot water
- Temperature probes
- Worksheet

Assumption of Prior Knowledge

- Students should have completed Algebra II/Trig
- Students should have basic understanding of Excel
- Students should be operating on Deduction level on Van Hiele scale with respect to data analysis.
- This lesson draws on the idea that no one wants to burn their tongue when they take that first sip from a hot beverage.

Introduction: Setting Up the Mathematical Task

In this lesson, students will identify an appropriate mathematical model for cooling behavior. Students will identify key characteristics for exponential and logarithmic graphs.

- This lesson will take one 90-minute block.
- Have any of you ever taken a sip from a hot beverage and burned yourself? What are ways you have tried to prevent yourself from getting burned?
- The students will answer the questions on the handout and share their findings.

Student Exploration:

Small Group Work: The teacher will boil a pot of water; pour the boiling water into each container. Each group will be responsible for collecting the data from one cup by recording the temperature every two minutes.
Whole Class Sharing/Discussion: Each group will share their individual data with the rest of the class.
Student/Teacher Actions:

- Encourage the students to research what the scalding temperature would be (use this as the starting temperature) and what the safe temperature is.
- Teacher will guide students as needed if questions/problems arise.

Monitoring Student Responses

- Have students discuss the worksheet in groups.

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Assessment

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a completed work grade out of 10 points - 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

Extensions and Connections (for all students)

What are other examples of exponential/logarithmic functions?

Strategies for Differentiation

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student exploration questions.
- Visual learners will benefit from the graphical representations within all the lessons.
- Auditory learners with benefit from the classroom and group discussions.
- High ability students may start to pick up on key features when creating graphs and to decide on what an appropriate model would be.

Name: _____ Date: _____

Cooling Temperatures

1. What type of mathematical model do you predict would be the most appropriate to represent the relationship between time and temperature? Why?

2. Record your data points (more tables on the back)

TIME (t)	TEMPERTURE (T)

3. Which variable will you represent as the independent/dependent variable? Why?

4. After graphing the data, what type of graph do you now feel is appropriate for the data?

5. $120°$ is considered a safe temperature to drink from. How long until this temperature is reached?

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Type of cup: _____

TIME (t)	TEMPERTURE (T)

Type of cup: _____

TIME (t)	TEMPERTURE (T)

Type of cup: _____

TIME (t)	TEMPERTURE (T)

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Name: _____ Date: _____

Cooling Temperatures –Exit Slip

1. What temperature would your beverage eventually reach? What is the mathematical term for this temperature?

2. What is the inverse of exponential decay?

3. What type of graph would best represent the heating of a liquid and its subsequent temperature measurement?

4. What are key features of an exponential function?

5. What are key features of a logarithmic function?

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Lesson 4 How Much Daylight Do We Get?

Strand
Data Analysis

Mathematical Objective(s)
Students will model periodic behavior by displaying the time of sunset (in hours past midnight), the time of sunrise (in hours past midnight), and total amount of sunlight (in hours) for an entire year. The students will need to research this information from the US Naval Observatory and enter this data into a spreadsheet, as well as create scatterplots and formulas. Students will then use this information to answer questions.

Mathematics Performance Expectation(s)
MPE.2. Students will collect and analyze data, determine the equation of the curve of best fit, make predictions and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

Related SOL

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

PH.2 The student will investigate and understand how to analyze and interpret data. Key concepts include

a) a description of a physical problem is translated into a mathematical statement in order to find a solution;
b) relationships between physical quantities are determined using the shape of a curve passing through experimentally obtained data;
c) the slope of a linear relationship is calculated and includes appropriate units;
d) interpolated, extrapolated, and analyzed trends are used to make predictions;
e) situations with vector quantities are analyzed utilizing trigonometric or graphical methods

NCTM Standards
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Lesson requires computers connected to the Internet and loaded with a spreadsheet program such as Excel.
- Websites to be used:
  http://aa.usno.navy.mil/data/docs/RS_OneYear.php
  http://aa.usno.navy.mil/data/docs/Dur_OneYear.php
  Students can import data for an entire year into Excel. There are instructions on how to import the data into Excel at the bottom of each page.

Assumption of Prior Knowledge

- Students should have completed Algebra II/Trig
- Students should have basic understanding of Excel
- The relevant real life context in this problem is related to the predictability of the length of days.

Introduction: Setting Up the Mathematical Task

In this lesson, students will investigate the properties of the sine and cosine functions using their graphs.

- This lesson should take two 90-minute blocks.
- To introduce the lesson, the students will look at an animation/video of a Ferris wheel.
  http://www.geogebratube.org/student/m34445
- While watching the video, choose a car (point) to focus on.
- As each second passes, the height changes.
- Class Discussion Questions:
  What would this scatterplot look like?
  Could a known function be used to find the car's (point's) position at any point in time?
  How could this function be used to find the times when the light is at its maximum and minimum heights?
- At the end of the student activity, the students will present their findings.

Student Exploration:

Group Work (groups of 2 or 3)

Whole Class Sharing/Discussion: At the beginning of the activity, the students will discuss what they expect their scatterplot to look like. They will make predictions of their outcomes.

Whole Class Sharing/Discussion: At the end of the activity, the students will share their outcomes and discuss whether their predictions matched their findings.

Student/Teacher Actions:

- Each group will have a computer.
- Students should begin by choosing a location to collect data from. The students will also need to decide if they want to choose sunrise, sunset, or day length.
- The students will enter this data into their spreadsheet by importing the data into Excel

- The teacher should walk around the room, monitoring student progress, and helping when needed. Depending on the level of knowledge of spreadsheets, some students may initially require more guidance than others.

Monitoring Student Responses

- Students should be discussing the questions:
    1. What shape does your graph have?
    2. Based on your data, what is the amplitude, period, phase shift and vertical shift for the function.
    3. What is the equation of the trig function with the amplitude, period, phase shift, and vertical shift that your graph has? (Hint: figure out what the four constants a, b, c, and d should be)
    4. What does your equation predict for today?
    5. What does your data tell you should actually be true today?
    6. Is the model equation you have inferred a good model for the data you have collected? Explain why or why not?

Assessment

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a completed work grade out of 10 points - 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

Extensions and Connections (for all students)

Think of another example that is periodic. What makes this periodic? Collect data and create an equation to help show that this example is periodic.

Strategies for Differentiation

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student exploration questions.
- Visual learners will benefit from the graphical representations within all the lessons.
- Auditory learners with benefit from the classroom and group discussions.
- High ability students may start to pick up on key features when creating graphs and to decide on what an appropriate model would be.

Name: _____ Date: _____

How Much Daylight Do We Get? –Exit Slip

1. Reflecting back to the Ferris Wheel: if the size were different (smaller/larger), how would this change the scatterplot?

2. How would our location change the scatterplot (for the activity)?

3. What are key features of periodic functions?

4. Describe how to find the equation of a periodic function.

5. An ecologist studying a species of water beetle estimates the population of a colony over an eight week period. If t is the number of weeks after the initial estimate is made, then the population is thousands can be modeled by $P(t) = 5 + 2\sin(60t)$ where $0 \leq t \leq 8$.
   a. What was the initial population?

   b. What were the smallest and largest populations?

   c. During what time interval(s) did the population exceed 6000?

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Lesson 5 Sum It Up

Strand
Algebra, Data Analysis

Mathematical Objective(s)
Students will brainstorm to come up with a real world situation which has data that is linear or exponential (or any type of relationship studied in the unit). Students will gather data (or find data online). Students will analyze data to determine the type of function that describes the data. Students will enter the data into a spreadsheet, plot it and find the equation of the curve of best fit.

Mathematics Performance Expectation(s)
MPE.2. Students will collect and analyze data, determine the equation of the curve of best fit, make predictions and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

Related SOL

AII.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

NCTM Standards
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources
- Lesson requires computers with Mystery Functions Activity (Excel)
- "Mystery Function" Handout
- "Real World Data Project" Handout
- Click on the icon to access the spreadsheet file.

Assumption of Prior Knowledge
- The students should have mastered the previous activities in this unit.

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Introduction: Setting Up the Mathematical Task

- In this lesson, students will review the types of functions covered in the previous lessons.
- This lesson should take one 90-minute block.
- To introduce the task, the students will be asked to recap what they have learned or important topics from the previous lessons.
- Students should be working in groups of 2 or 3.

Student Exploration:

Small Group Work: The teacher put students in groups of 2 or 3

Whole Class Sharing/Discussion: At the beginning of the activity, the class should briefly discuss characteristics of the different functions studied all week. They should complete the mystery function activity and answer the questions on the "Mystery Function Handout"

Student/Teacher Actions:

- Each student or group of students should have a computer to work on the Mystery Function Activity.
- The students will be attempting to establish the equation of the function. The students may be able to establish the equation based on the different points. They may need to create scatterplots to help determine what type of function they are working with.
- The teacher should walk around the room, monitoring student progress, and helping when needed.
- The students will answer the questions on the "Mystery Function" Handout.
- This activity should take about 20 minutes – it is meant to be an introduction to the final project.
- Answers: (Mystery Function Excel Activity)
  - Problem 1: $log_3 x$
  - Problem 2: $\sin\left(\frac{x}{2}\right) + 1$
  - Problem 3: $-3x - 4$
  - Problem 4: $-3(x-1)^2 + 1$
  - Problem 5: $100\left(\frac{1}{2}\right)^x$
  - Problem 6: $4\left(\frac{3}{2}\right)^x$
- When the introduction is complete, the students should receive the "Real World Data Project" Handout.
- It will be time saving if the students can find their data online and use that ready-made data to complete the project.

Monitoring Student Responses

- The students should discuss what type of function they are working with: linear, quadratic, exponential, logarithmic, periodic. They should explain why they came to their conclusions.

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Assessment

Students will complete a Real World Data Project in groups of 2 or 3. A rubric is included for assessment.

Extensions and Connections (for all students)

Students will be encouraged to brainstorm about transformations of the various graphs. Is there a set rule we can use for all graphs?

What features of the graphs should be established? Encourage the students to identify these:

- Domain/range
- Zeros
- X- and y-intercepts
- Asymptotes
- Maximum/minimum

Strategies for Differentiation

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student exploration questions.
- Visual learners will benefit from the graphical representations within all the lessons.
- Auditory learners with benefit from the classroom and group discussions.
- High ability students may start to pick up on key features when creating graphs and to decide on what an appropriate model would be.

Name: _____ Date: _____

Mystery Function

On your computer, open the Excel File called “Mystery Function”.

Write your answers to the Mystery Functions below.

1. 4.

2. 5.

3. 6.

Answer the following questions about the Mystery Functions.

7. What are key features of each function?

8. How did you derive the equations?

9. What are other ways to derive the equations?

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Real World Data Project Name_____Date_____

Now that you have reviewed all the types of functions we have looked at in this Unit, it's time to find your own data!!

In your group, brainstorm to come up with your own ideas for real-world examples of these types of functions. It will be time-saving if you can find data online instead of collecting it yourselves.

Once you have found your Real World situation and the corresponding data, complete the following tasks.

1. Enter the data into a spreadsheet. Make sure all data is correctly labeled. Format data to show 2 decimal places, and wrap text if necessary.
2. Make a scatter plot of your data and find the curve of best fit. (Make sure your scatter plot and equation of the curve are shown on the spreadsheet.)
3. Label the scatter plot correctly, and include a title.
4. Use or create a formula that makes sense for your data. (Sum, average,etc…). Show your formula in a cell on your spreadsheet.
5. Create a pie chart, column graph or other chart for your data. Label correctly and include on your spreadsheet.
6. Extra points will be awarded for choosing data which is something OTHER than linearly related.
7. Print your spreadsheet and turn in. ☺

	1 point	2 points	3 points
Enter and format data Create and/or use formulas	Data is entered and formatted with some mistakes. Formula is missing or incorrect.	Data is entered and formatted. Formula is used. There are some mistakes.	Data is entered and formatted correctly. Formula is used correctly.
Create a scatter plot and find equation of curve of best fit	Scatter plot is complete but not labeled correctly. Equation is incorrect or missing.	Scatter plot is complete and labeled. Equation is included. There are some mistakes.	Scatter plot is complete and labeled correctly. Equation is included and correct.
Create a Chart	Chart is included but not labeled.	Chart is included and labeled but with some mistakes.	Chart is included and labeled correctly.
Extra Points	Up to 2 extra points may be awarded for data that is non-linear.		

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